Conversions: Degrees to Radians, & Degrees° Minutes’ Seconds” to decimal degrees

Remember: 1 radian = \( \frac{180}{\pi} \) degrees and 1 degree = \( \frac{\pi}{180} \) radians & 1° = 60 minutes = 3600 seconds, because 1 min = 60 sec

Convert each degree measure to radians. Leave answers in terms of \( \pi \). Then convert each radian measure to degrees.

1. a) 315° b) -240° 2. a) \( \frac{4\pi}{3} \) b) \( \frac{11\pi}{6} \)

<table>
<thead>
<tr>
<th>Convert each measure to Degrees° Minutes’ Seconds”</th>
<th>Convert each D°M’S” to decimal degrees. Express answers to the nearest hundredth (x.xx).</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. 150.7863°</td>
<td>4. 1°20’ 40”</td>
</tr>
<tr>
<td>4. 240.83°</td>
<td>5. 134°25’42”</td>
</tr>
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</table>

The Unit Circle

Find the Reference Angle

6. -565° 7. 410° 8. \( \frac{4\pi}{3} \) 9. \( \frac{11\pi}{6} \)

Determine the Quadrant that \( \theta \) is located in;

10. \( \sin \theta > 0 \) and \( \tan \theta < 0 \) 11. \( \sec \theta > 0 \) and \( \csc \theta < 0 \)

Give the EXACT value of each of the following:

12. \( \sin 330° \) 13. \( \cos \left( \frac{\pi}{2} \right) \) = 14. \( \tan \left( \frac{7\pi}{2} \right) \) = 15. \( \sec \left( \frac{3\pi}{2} \right) \) = 16. Csc 240°

sin and cosine graphs

Remember: \( y = A \sin C(x - s) + L \) and \( y = A \cos C(x - s) + L \)

Graph two cycles of each equation (one cycle to the right and one cycle to the left of the y-axis). Label the coordinates of the following: any nodes, one maximum point and one minimum point.

17. \( y = 3\cos x - 2 \) 18. \( y = 3\sin(x + \frac{\pi}{2}) - 3 \) 19. \( y = 2\cos(x - \frac{3\pi}{4}) - 2 \) 20. \( y = 3\csc 2x \)
Pythagorean Theorem & Right Triangles

Remember the Pythagorean Theorem:

\[ \text{Leg}^2 + \text{Leg}^2 = \text{Hyp.}^2 \]

& SOH CAH TOA

\[ \sin \theta = \frac{\text{opp. side}}{\text{hypotenuse}}, \quad \cos \theta = \frac{\text{adj. side}}{\text{hypotenuse}}, \quad \tan \theta = \frac{\text{opp. side}}{\text{adj. side}} \]

For each problem, provide a neat, well-labeled sketch of the situation described. Show your work on a separate sheet of paper in a neat and organized manner. Round all answers to the nearest tenth of a unit.

21. \( a = 10; \; b = \_\_\_; \; c = 26 \)

22. \( a = 11; \; b = \_\_\_; \; c = 61 \)

23. \( a = 6; \; \beta = 40^\circ; \; \text{find } b, c, \text{ and } \alpha \)

24. \( a = 7; \; \beta = 50^\circ; \; \text{find } a, c, \text{ and } \alpha \)

25. \( c = 9; \; \beta = 20^\circ; \; \text{find } a, b, \text{ and } \alpha \)

26. A ladder is 25 ft. long. The ladder needs to reach to a window that is 24 ft. above the ground. How far away from the building should the bottom of the ladder be placed?

27. The angle of elevation from the control tower to an airplane is 49°. The airplane is flying at 5000 ft. How far away from the control tower is the plane?

OBLIQUE TRIANGLES

Remember the Law of Sines and

\[ \frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} \]

Law of Cosines

\[ a^2 = b^2 + c^2 - 2bc \cos \alpha \]

\[ b^2 = a^2 + c^2 - 2ac \cos \beta \]

\[ c^2 = a^2 + b^2 - 2ab \cos \gamma \]

Use the Law of Sines when you know an angle and the length of the side opposite it. Use the Law of Cosines when you have two sides and an “included” angle (SAS) or when you know the length of three sides (SSS).

28. \( a = 5; \; \alpha = 45^\circ; \; \beta = 95^\circ \)

29. \( b = 5; \; \alpha = 45^\circ; \; \beta = 50^\circ \)

30. \( c = 6; \; \gamma = 8; \; \alpha = 50^\circ \)

31. \( \alpha = 40^\circ; \; \beta = 20^\circ; \; a = 2 \)

32. \( a = 2; \; c = 1; \; \beta = 10^\circ \)

33. \( a = 9; \; b = 7; \; c = 10 \)

34. Curly needs to determine the height of a tree before cutting it down to be sure that it will not fall on a nearby house. The angle of elevation to the tree from one position on a flat path from the tree is 30°, and from a second position 40 feet farther along the path is 20°. What is the height of the tree?

35. A ship at sea is sighted by two different observation posts, A and B on shore. Points A and B are 24 kilometers apart. The measure of the angle at A between \( \overline{AB} \) and the ship is 41.6°. The angle at B is 36.1°. Find the distance to A to the ship.
Area of Triangles

Use: Area = \( \frac{1}{2} \) ab sin C, Area = \( \frac{1}{2} \) bc sin A or Area = \( \frac{1}{2} \) ac sin B

OR Heron’s Formula: Area = \( \sqrt{s(s-a)(s-b)(s-c)} \) where \( s = \frac{1}{2}(a+b+c) \)

find the area of each triangle. Round answers to two decimal places.

36. a = 3; b = 4; \( \gamma = 40^\circ \)
37. a = 2; c = 1; \( \beta = 10^\circ \)
38. a = 2; b = 3; c = 2
39. a = 5; b = 8; c = 9

Bearings & Vectors

Bearings are ALWAYS measured CLOCKWISE FROM NORTH, like these shown here:

Bearing 060°  Bearing 240°  Bearing 330°

And with “Vectors” the phrase that pays:

“TIP TO TAIL AND YOU’LL NEVER FAIL”.

That’s how you “add” Vectors

Solve these with either the “Geometric Method” (using the Law of Cosines & Law of Sines) or the “Component Method” (using right triangle trig).

40. The River’s velocity is 10 mi/hr at a bearing of 030° and the Boat travels at 5 mi/hr @ 270°.

41. The River’s velocity is 7 mi/hr at a bearing of 145° and the Boat travels at 5 mi/hr @ 305°.

43. – Force A = 10 lbs.  Angle A = 20°
    Force B = 22 lbs.  Angle B = 10°
    Force C = 7 lbs.  Angle C = 20°

Vectors can also be written in a “positional” notation, which deals with the vector’s direction, but since a vector MUST also be defined by it’s magnitude, we must use a method to determine the magnitude of a given vector in positional notation. To do this, simply use the Pythagorean Theorem.

ie: \( \vec{V} = \langle 3, -4 \rangle \), then the “magnitude” of vector \( \| \vec{V} \| = \sqrt{3^2 + (-4)^2} = 5 \)

Try finding the magnitude of these vectors:

44. \( \langle 3, 5 \rangle \)
45. \( \langle 7, -3 \rangle \)

Add the vectors and give the resultant answer in “positional” notation along with the Resultant’s Magnitude:

46. \( \langle -7, 5 \rangle + \langle 2, 4 \rangle \)
47. \( \langle -3, -5 \rangle - \langle -2, -4 \rangle \)
Polar Coordinates

Remember – Polar coordinates are given by the “Radius and Angular direction \((r, \theta)\)
just like a vector, it has a magnitude (radius) and the direction.

Convert these rectangular coordinates to Polar coordinates:
48. \((3, -5)\) 49. \((5, 4)\)

Convert these Polar coordinates to Rectangular coordinates:
50. \((3, 27^\circ)\) 51. \((7, 120^\circ)\)

Trigonometric Identities & Equations

Remember that an identity is an expression that is always true for any value of \(\theta\). Here are few key identities you should remember from your “cliff notes” page created during that unit.

\[
\begin{align*}
csc \theta &= \frac{1}{\sin \theta} & \sec \theta &= \frac{1}{\cos \theta} & \tan \theta &= \frac{\sin \theta}{\cos \theta} & \cot \theta &= \frac{\cos \theta}{\sin \theta} & \sin^2 \theta + \cos^2 \theta &= 1 \\
\end{align*}
\]

Substitute the fundamental, reciprocal, or quotient identity and simplify with algebra where necessary.

52. \(1 - \cos^2 x\) 53. \(\sin \theta + \cos \theta \cdot \cot \theta\) 54. \(1 - \sin^2 x\) 55. \(\frac{1}{\cos^2 \theta} - 1\)

56. \(\tan \phi + \cot \phi\) 57. \(\frac{\cos \alpha}{1 + \sin \alpha} + \frac{\sin \alpha}{\cos \alpha}\)

Solve the equation for the angle \(\theta\) such that \(0^\circ < \theta < 360^\circ\)

58. \(\sin x + 1 = 0\) 59. \(\cos^2 x + \cos x - 2 = 0\) 60. \(1 = \sin(x - \frac{\pi}{4})\)